Linear Algebra – MATH-4100
Syllabus for Fall 2009

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Office Hours: T & TΘ 14:00-15:00 & by appointment

Text Linear Algebra Done Right (2nd edition)
by Sheldon Axler
Springer Verlag, 1997 (2004 corrected printing).

Grades

- A: 90-100, A-: 85-89, B+: 82-84, B: 78-81, B-: 75-77, C+: 72-74, C: 68-71, C-: 65-67, D+: 62-64, D: 58-61, F: <58
- Homework: 25% (5% penalty for handing in late)
- Tests 1 & 2: 25% each
- Test 3: (a formal final exam): 25%

9/1 Background: functions, domain, range, injective, surjective bijective
Fields: $\mathbb{F} = \mathbb{R}$ or $\mathbb{C}$; properties, inverse and norm in $\mathbb{C}$
Vector spaces: axioms and simple consequences
Subspaces: definition and criteria
Examples of vector spaces and subspaces, including power series and polynomials

9/4 Sums and direct sums of subspaces
lists of vectors (not sets), linear combination, span, finite & infinite dimensional, linear independence & dependence

9/7 Chapter 2: 2.4-2.7
Chapter 2: Bases up to 2.13

9/11 A fundamental invariant of a vector space: Dimension

9/14 Linear maps (linear transformations) & operators
null space and range of linear maps and operators

9/18 Theorems on null spaces & ranges of linear maps
linear equations: m equations in n unknowns; homogeneous and inhomogeneous forms

9/21 Matrices: Definitions, relation to linear maps
one operator, many matrices

9/25 Invertibility: Theory in finite dimensions, exceptions in infinite dimensions

9/29 Invertibility: matrices, change of basis, elementary transformations

10/2 Polynomials: long division, roots over $\mathbb{C}$; factoring, roots over $\mathbb{R}$; polynomials applied to operators

10/6 In class test.

10/9 Eigenvalues and eigenvectors of linear operators, invariant subspaces
Basic theorems and examples in finite & infinite dimensions

10/16 UT (upper triangular matrices): Fundamental theorem for linear operators on finite dimensional complex vector spaces (two stage proof; profound)

10/20 Continuation of UT matrices and relation to linear maps and bases
Diagonal matrices

10/23 Invariant subspaces for linear operators over real vector spaces
$V \neq \text{null}(T - \lambda I) \oplus \text{range}(T - \lambda I)$, but $V = \text{null}(T - \lambda I)^n \oplus \text{range}(T - \lambda I)^n$. The Jordan canonical form.

10/27 IP (Inner product) spaces: basic results to 6.14
Examples, including orthogonal polynomials

10/30 ON (orthonormal) bases, OTH (orthogonal bases), Gramm-Schmidt algorithm (Zuker method), up to 6.28

11/3 OP (orthogonal projection) and minimization problems ("least squares")

11/6 Linear functionals and adjoints
theory & examples in finite dimensions and an example in infinite dimensions

11/10 More on IP spaces & solutions to problems 30-32 (Chapter 6)

11/13 In class test

11/17 Introduction to self-adjoint and normal operators
Basic theory

11/20 The complex spectral theorem: A normal operator over a finite dimensional complex IP space has a basis of eigenvectors; equivalent statements

11/24 The real spectral theorem, (skip normal operators on real IP spaces except to give results and "hand wave" why the result is "obvious")

12/1 Positive operators and basic results

12/4 Isometries (rotations) and relation to skew symmetric operators.

12/8 Polar & SV (singular value) decomposition

12/11 Finish topics on positive operators; linear maps transform the n-dimensional sphere into an n-dimensional ellipsoid; norm of operator/matrix