Problem Set 6

Chapter 5 and extra.

Note: Z1, Z2 etc. refer to my own problems, which may be very close to what is in the text.

Problems 4, 6, 7, 9 and 11 on pages 94 & 95.

Z1. Suppose that $P \in \mathcal{L}(V)$ satisfies $P^2 = P$. Compute all possible eigenvalues of P.

Z2. Suppose that $T \in \mathcal{L}(V)$ satisfies $T^2 = 0$.

- Compute $(I + T)^2$.
- Compute $(I + T)^n$ for any $n > 0$.
- Compute all the eigenvalues of T.
- Compute all the eigenvalues of $I + T$.
- If $V = \mathbb{R}^m$ and t is any real number, compute $e^{t(I+T)}$. Note that
$$e^S = \sum_{i=0}^{\infty} \frac{S^i}{i!}$$
for any $S \in \mathcal{L}(\mathbb{R}^m)$.
- Compute e^{tA}, where t is a real number and
$$A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}.$$

Z3. Let $D \in \mathcal{L}(\mathbb{F}^{\infty}[x])$ be the derivative operator on power series. That is
$$D\Big(\sum_{i=0}^{\infty} a_i x^i\Big) = \sum_{i=0}^{\infty} (i+1) a_{i+1} x^i.$$

- Compute all eigenvalues of D.
- If λ is an eigenvalue of D, compute an eigenvector corresponding to it.
- If λ is an eigenvalue of D, compute a basis for $\operatorname{null}(D - \lambda I)$. What is $\dim \operatorname{null}(D - \lambda I)$?