Human Population
2017

Lecture 4
Exponential Growth
Systems dynamics
Age demographics
Questions from the reading?
The goal is to model
...human population as a function of time.

Items needed to build a model:
1) data
2) an understanding of the data.
3) elements of modeling
   A. equations
   B. feedback
   C. sensitivity
Thinking critically
...about modeling humanity.

Can we model humans as a population? (i.e. a single entity)

Can we model any animal population as a single entity? Is it a valid model? Is it relevant?

Does human intelligence mean that humans cannot be modeled?
Linear growth. Constant increase or decrease. Makes sense for lots of systems, but not for population growth since a constant number of births requires a ever-decreasing birth rate.
Model 1. Exponential growth

\[ \frac{dN}{dt} = \alpha N - \beta N = (\alpha - \beta)N \]

Population growth is proportional to population. This is a natural population growth curve for the case of:

- no predation,
- no shortages,
- no space limitations.
Total number of rice grains received after filling in all 64 squares: 18,446,744,073,709,551,615

Number of rice grains in a ton of rice: 64,000,000

Tons of rice awarded: 288,218,750,000

Size of cooking pot large enough to hold this much rice: 5 mi wide, 5 mi tall.
A water lily plant doubles in size every day, covering the pond in 30 days. When was the pond half-filled with water lillies?
negative exponential growth of pond size

Often when we see exponential decline, (not decay) we are talking about negative exponential growth.
Things that grow exponentially...

BACTERIAL GROWTH

Food to mass ratio decreases

LAG PHASE  LOG PHASE  DECLINING GROWTH PHASE  ENDOGENOUS RESPIRATION PHASE

Bacterial Growth Curve  Food Curve

Time  Bacterial Mass / Amount of Food
Things that grow exponentially...

Spread of ebola

at least for awhile...

\( \sim e^{0.028x} \)

Data: http://en.wikipedia.org/wiki/Ebola_virus_epidemic_in_West_Africa#Timeline_of_cases_and_deaths
Author: Geert Barentsen (geert.io / @GeertHub)
Things that grow exponentially...

Yeast grows rapidly on sugar, making bread and beer possible. Upon overshoot, yeast die equally quickly.
Reindeer isolated on an island with no predators, grew exponentially, consumed all of the vegetation to the roots, eliminating the power of the vegetation to regenerate, leading to a quick die-off.
Things that grow exponentially...

Irish

Carrying capacity for humans was maximized by employing a crop that produced a maximal number of calories per acre, the potato. Fungal phytopathogen *P. infestans* grew exponentially until it wiped out its food source, the same food as that of the Irish. Both populations collapsed.
The power of population is so superior to the power of the earth to produce subsistence for man, that premature death must in some shape or other visit the human race. The vices of mankind are active and able ministers of depopulation. They are the precursors in the great army of destruction, and often finish the dreadful work themselves. But should they fail in this war of extermination, sickly seasons, epidemics, pestilence, and plague advance in terrific array, and sweep off their thousands and tens of thousands. Should success be still incomplete, gigantic inevitable famine stalks in the rear, and with one mighty blow levels the population with the food of the world.

Malthus pointed out that populations grow exponentially (geometrically), but food production grows linearly (arithmetically).

Malthus was almost right...
Actually, food production is limited by arable land, which is finite.

Was Malthus almost right? Will exponential population growth will always overtake food production?
Malthusian oscillations in a limited exponential growth model

A simple simulation of exponential growth in a finite world of plants and animals.

Plants cannot exceed 100%.

Animals grow by a fixed growth factor, unless limited by plants.

Plants recover after a fixed delay time.

In this simulation, growth factor = 1.2.
Recovery time = 12.
the math of exponential growth

• When the rate of growth is proportional to the current amount, then growth is "geometric". That is, it increases by the same factor at each time period.

• For example: 1, 2, 4, 8, 16, 32, 64 or 1.01, 1.0201, 1.030301, etc

• \( N_t = N_0 g^t \), where \( t \) is time, \( N_0 \) is the starting amount, \( g \) is the "growth factor" per unit time.

• For example: \( 64 = 1 \times 2^6 \), \( 1.030301 = 1.01 \times (1.01)^2 \)
the math of exponential growth

- Converting to base-e...
- \( N_t = N_0 g^t \)
- Define growth rate: \( r = \ln(g) \)
- Then, \( r = \ln(\frac{N_t}{N_0}) / t \)
  The growth rate is the natural log of the average growth factor.
- \( N_t = N_0 \exp(r \cdot t) \)

<table>
<thead>
<tr>
<th>Growth Factor</th>
<th>Growth Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.69</td>
</tr>
<tr>
<td>1.5</td>
<td>0.41</td>
</tr>
<tr>
<td>1.1</td>
<td>0.095</td>
</tr>
<tr>
<td>1.01</td>
<td>0.0099</td>
</tr>
<tr>
<td>1.001</td>
<td>0.001</td>
</tr>
</tbody>
</table>

if \( r \ll 1 \), \( 1 + r \approx r \)

For slow growth, the rate equals the growth factor minus one.
doubling time

- \( T_d = \) doubling time = time when \( \frac{N_t}{N_0} = 2 \)
- \( r = \ln\left( \frac{N_t}{N_0} \right) / t \)
- \( T_d = \frac{\ln(2)}{r} \)

\[ \approx \frac{70}{g} \] ...where \( g \) is the % growth.
## Doubling Time Quiz

<table>
<thead>
<tr>
<th>Rate</th>
<th>Doubling Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%/year</td>
<td></td>
</tr>
<tr>
<td>1%/year</td>
<td></td>
</tr>
<tr>
<td>2%/day</td>
<td></td>
</tr>
<tr>
<td>0.5%/day</td>
<td></td>
</tr>
</tbody>
</table>
negative growth: half-life

- $T_{1/2} = \text{half-life} = \text{time when } N_t/N_0 = 1/2$
- $r = \ln\left(\frac{N_t}{N_0}\right) / t$
- $T_d = \ln(1/2)/r = -\ln(2)/r$

$\approx \frac{70}{g} \quad \text{...where } g \text{ is now the } \% \text{ decrease.}$
# Doubling Time Quiz

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</table>
Types of feedback cycle

- $N_{t+1} = N_t + f(N_t)$

- Examples:
  - $N_{t+1} = N_t + \alpha N_t$ \textit{positive feedback}
  - $N_{t+1} = N_t - 1/2 N_t$ \textit{negative feedback}
  - $N_{t+1} = N_t + e^{N_t}$ \textit{positive feedback}
  - $N_{t+1} = N_t + 1/N_t$ \textit{negative feedback}
Exp. growth has positive feedback. Exp. decay has negative feedback.

\[ N_{t+1} = N_t + \alpha N_t - \beta N_t \]

\[ N_t = N_0 \exp(r \cdot t) \]

• \( N_t = N_0 \exp((\alpha - \beta) \cdot t) \)

where \( \alpha \) is the birth rate and \( \beta \) is the death rate.
Growth with negative feedback

**Limited linear growth**

\[ N_{t+1} = N_t + \alpha (N_{\text{max}} - N_t) \]

**Limited exponential growth**

\[ N_{t+1} = N_t + \alpha N_t (N_{\text{max}} - N_t) \]

**Unlimited, always slowing**

\[ N_{t+1} = N_t + \frac{1}{t} \]

\[ N_t = \log(t) \]

**Unlimited, slowing slowly**

\[ N_{t+1} = N_t + \frac{1}{N_t} \]
Human population growth is not a "single-exponential"

In the best fit to a single exponential, the population would have been 2 in the year 500!

Population growth was super exponential between 1000 and 1980. The doubling time decreased from 700 years to 130 years to 40 years.

"Single" exponential growth looks like a line when plotted in log-scale.

Human growth rate changed dramatically over history.
The past 12 thousand years

Population growth was super exponential between 1000 and 1980. The doubling time decreased from 700 years to 130 years to 40 years over 1000 years.

Log-linear segments show periods of constant exponential growth/decline.

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https://www.census.gov/population/international/data/worldpop/table_history.php
Why did population grow erratically?

1. Disease.
2. Technology.
3. Climate.
4. Disasters.
5. Regional differences.
6. Competition.
What sort of historical events correlate with increase/decrease in growth rate?

http://worldpopulationhistory.org/
Part 1: make a model for population

people = 100
birth = [birth rate]*[people]
death = [death rate]*[people]
birth rate = slider from 0 to 0.1
death rate = slider from 0 to 0.1
Results of a simulation

What if we want "error bars"?
Sensitivity testing in InsightMaker

• When the exact value of a variable is not known we test a distribution of values.

• Must add a random number generator in = window (value/equation window)
  • value menu > Random Number Functions > Normal Distribution
    Places RandNormal(Mean,Standard Deviation) in equation window. Randomizes every timestep, unless "Fix" is added, then only randomizes on first timestep.
    • Fix(RandNormal(Mean,Standard Deviation))
  • Replace "Mean" with a number or Variable.
  • Replace "Standard Deviation" with a number or variable.

– Tools>Sensitivity Testing
  – Set Primitives to monitor, number of runs, confidence regions to plot. Run analysis.
IM 2: sensitivity of a model for population

people = 100
birth rate = Fix(RandNormal([mean birth rate], [uncertainty in birth rate]))
death rate = Fix(RandNormal([mean death rate], [uncertainty in death rate]))
other variables are sliders.
This says "my estimates of error in Variables A and B translate to this estimate of error in Stock Y (people)."
Age Demographics

Population may be broken down by age and sex, sometimes called "population pyramids, since they get smaller as they go up. Age demographics are useful for predicting future growth/decline and for comparing between nations or regions.
WORLD 2016
Population: 7,432,663,000

http://populationpyramid.net/
Japan's birth rate and death rate both became low after WW2 with the subsequent economic and quality of life improvements.
Age demographics of Europe

http://www.viewsfromtheworld.net/
Family from growing population

photo from blogs.redcross.org.uk.

age pyramid for Mali 2015
Family from shrinking population

chinese family
http://populationpyramid.net/
Stock type: conveyer

Number In Overflow bucket = [people]
Total Number In all buckets = [[[people]]]

If you plot a conveyer stock, you see only the Overflow. Create a variable and link it to [[stock]], plot that instead.

Read about Stocks:  https://insightmaker.com/stocks
IM 3: age demographics using "conveyer" stocks

\[
\text{child} = 100 \quad \text{(delay}=25) , \quad \text{adult} = 50 \quad \text{(delay}=30) , \quad \text{old} = 20 , \quad \text{people}=[\text{child}]+[\text{adult}]+[\text{old}] \\
\text{mean birth rate} = \text{Fix} (\text{RandNormal}([\text{mean birth rate}], [\text{uncertainty in birth rate}])) \\
\text{death rates} = \text{sliders.} \\
\text{simulate for 200 years. Do sensitivity test.}
Age demographics of a shrinking population

Year | 0  | 25  | 55  | 80  | 150
---|---|---|---|---|---
Child | | | | | 
Adult | | | | | 
Old | | | | |
Age demographics of a growing population
What determines age demographics?

• past birth rates, maternal ages
• past death rates, ages at time of death
• immigration, emigration by age group
• wars, epidemics
• child health, maternal care
• fertility, family planning
What use is age demographics?

- predicts future growth
- reflects current fertility/mortality
- may relate to economics
- determines "inertia" in growth
- may show sex-biased abortion, infanticide.
Model 2. Logistic growth

Demographic Transition Model
Carrying capacity
Van Foerster's Doomsday
InsightMaker primitive: converter

- A Converter takes Variables and applies a function to them, outputting a value.